Viscoelastic characterization of soft tissue from
dynamic finite element models

Hani Eskandari¹, Septimiu E Salcudean¹, Robert Rohling¹
and Jacques Ohayon²

¹ Department of Electrical and Computer Engineering, University of British Columbia,
Vancouver, BC, Canada
² Laboratoire TIMC/IMAG, Faculty of Medicine, Université Joseph Fourier de Grenoble,
La Tronche, France

E-mail: hanie@ece.ubc.ca, tims@ece.ubc.ca, rohling@ece.ubc.ca and jacques.ohayon@imag.fr

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Abstract
An iterative solution to the inverse problem of elasticity and viscosity is proposed in this paper. A new dynamic finite element model that is consistent with known rheological models has been derived to account for the viscoelastic changes in soft tissue. The model assumes known lumped masses at the nodes, and comprises two vectors of elasticity and viscosity parameters that depend on the material elasticity and viscosity distribution, respectively. Using this deformation model and the observed dynamic data for harmonic excitation, the inverse problem is solved to reconstruct the viscosity and elasticity in the medium by using a Gauss–Newton-based approach. As in other inverse problems, previous knowledge of the parameters on the boundaries of the medium is necessary to assure uniqueness and convergence and to obtain an accurate map of the viscoelastic properties. The sensitivity of the solutions to noise, model and boundary conditions has been studied through numerical simulations. Experimental results are also presented. The viscosity and elasticity of a gelatin-based phantom with inclusion of known properties have been reconstructed and have been shown to be close to the values obtained using standard rheometry.

1. Introduction
Malignant tumors and carcinomas have different mechanical properties compared with normal tissue. For many years, physicians have located tumors in soft human tissues by palpating the patient. In the last two decades, many studies have revealed the significance of tissue elasticity (e.g. Ophir et al 1996, Pellot-Barakat et al 2006) and viscosity (e.g. Sinkus et al 2005a, 2005b) in classifying normal, benign or malignant masses. Several methods have
been developed to estimate the local parameters in the viscoelastic medium of soft tissues. Generally, via an imaging modality such as ultrasound or magnetic resonance imaging (MRI), the internal motion of the body is estimated in response to an excitation and can be analyzed to reconstruct the local variations in the viscoelastic properties of tissue. Surveys of such methods can be found in Ophir et al. (1996), Liu et al. (2005), Pellot-Barakat et al. (2006), Sridhar et al. (2007) and Eskandari et al. (2008).

Ultrasound and MRI are the most popular imaging modalities for tracking tissue deformation due to an excitation. The reconstruction of the elasticity from the boundary force and the internal displacements measured with ultrasound elastography can be formulated in several ways. While a three-dimensional (3D) modeling scheme is the most realistic, traditional 2D ultrasound imaging (the most common approach) has limitations on the accuracy of measuring the three components of motion. Axial displacements are measured with the highest accuracy, lateral displacement measurement is less accurate and elevational displacement measurement is often impossible (note that in ultrasound and ultrasound elastography, axial is along the direction of the excitation and pulse propagation, lateral is perpendicular to axial but within the imaging plane, and elevational is perpendicular to the imaging plane). These limitations have given rise to techniques that simplify the model or ignore one or two components of the displacements. If we consider only the axial displacement and a one-dimensional (1D) model, the reciprocal of the quasi-static axial strain can be interpreted as the local elasticity. The viscosity or the relaxation time can be estimated by using a model that predicts the displacement or strain as a function of frequency (Insana et al. 2005, Turgay et al. 2006, Eskandari et al. 2008, Sridhar et al. 2007). The moduli reconstructed using 1D models suffer from artifacts, because the effect of the boundary conditions on strain is not decoupled from the effect of material properties.

To account for axial and lateral displacements, the partial differential equations that describe the stress–strain relationship should be evaluated in a plane. Assumptions on the deformation profile such as being in a plane-stress or plane-strain state can help simplify the general 3D equations. Furthermore, assuming a nearly incompressible and isotropic medium significantly reduces the number of elasticity parameters and leaves Young’s modulus as the only unknown. If the problem is discretized and formulated such that Young’s moduli are the unknowns, with known displacements and strains, a forward solution technique can be applied to estimate the parameters (Raghavan and Yagle 1994, Sumi et al. 1995, Skovoroda et al. 1995, Bishop et al. 2000, Zhu et al. 2003, Park and Maniatty 2006). Most of the methods in this category require the 2D displacement and strain fields as well as their first and second derivatives in the region of interest. However, due to the low lateral resolution of ultrasound, the estimated lateral displacements have low signal-to-noise ratios (SNRs). Once the low SNR of the lateral displacements has been overcome and the boundary forces have been measured, a reliable elasticity distribution may be obtained (Park and Maniatty 2006).

In another approach, the inverse problem of elasticity is solved in the sense that a specific functional is minimized (Kallel and Bertrand 1996, Fu et al. 2000, Miga 2003, Oberai et al. 2004, Doyley et al. 2004, Liu et al. 2005, Khalil et al. 2005). Usually, this functional is considered to be the quadratic norm of the difference between the measured displacements and the displacements resulting from an assumed distribution of elasticity. Using the finite element method (FEM), iterative strategies based on Gauss–Newton or quasi-Newton methods have been proposed in the literature. Also, a method has been suggested to evaluate the gradient using adjoint equations in order to increase the computational efficiency (Oberai et al. 2004). The exact analytical solutions for the inverse problem of elasticity and other quasi-static parameters have been investigated in Barbone and Oberai (2007), where it was shown that the problem of determining the shear modulus becomes unstable for nearly
incompressible materials. It was shown that the uniqueness of the solution is highly dependent on the regularization, available boundary conditions, prior knowledge of the elasticity on the boundaries and the incompressibility of the medium (Barbone and Bamber 2002, Barbone and Gokhale 2004, McLaughlin and Yoon 2004). Recently, the authors have shown that applying wide-band excitation and calculating the asymptotic low-frequency magnitude of the transfer functions can increase the accuracy of the elasticity estimations (Eskandari and Salcudean 2006).

Estimation of the viscosity, however, requires applying dynamic excitation to the material and calculating higher order derivatives (material velocities). Hence, compared with the elasticity reconstruction, the viscosity estimates will be more sensitive to noise. Viscoelastic modeling and parameter identification have also been explored extensively in the literature (Walker et al 2000, Chen et al 2004, Catheline et al 2004, Bercoff et al 2004, Insana et al 2005, Sinkus et al 2005a, Girnyk et al 2006, Turgay et al 2006, Sridhar et al 2007, Eskandari et al 2007). Basically, through analysis of the attenuation and phase change of the propagating waves or the relaxation behavior of the structure, the dynamic properties can be characterized.

The problem of reconstructing the damping matrix in a dynamic FEM framework has been mostly studied in the context of structural dynamics (Gontier et al 1993, Pilkey et al 1999, Adhikari 2002), where the goal is to identify an overall damping matrix for a given structure.

In this work, the inverse problem of reconstructing the stiffness and viscosity using a dynamic finite element approach is tackled. A model is proposed that incorporates the Voigt model for the viscoelastic deformation of soft tissues in a dynamic finite element formulation. The inverse problem of viscosity and elasticity is solved based on harmonic measurements of the axial displacements. The algorithm is devised such that by knowing the parameters on the boundaries, their distribution can be estimated inside the medium, without needing to know the force on the boundaries or the lateral displacements in the region of interest. The method is similar to the elasticity reconstruction algorithm by Kallel and Bertrand (1996), in the sense that a displacement functional is minimized using a Gauss–Newton approach. However, the problem has been reformulated to account for the viscous damping effect, unknown lateral displacements and effects of the unknown forces on the Jacobian matrix. The accuracy of the proposed algorithm is evaluated using dynamic finite element simulations. Preliminary studies have been performed on tissue-mimicking phantoms with embedded elasticity and viscosity inclusions. Gelatin phantoms with controlled proportion of gelatin powder are widely used to model the mechanical and ultrasonic characteristics of human soft tissues. The estimation results are compared with other parameter identification techniques and rheometry data.

2. Model

2.1. Linear viscoelastic model

In this paper, normal fonts denote scalar parameters while vectors are shown in lowercase bold and matrices in uppercase bold.

The overall stress tensor \( \sigma_{ij} \) for a viscoelastic material can be divided into elastic and viscous terms as follows (Malvern 1969):

\[
\sigma_{ij} = \sigma_{ij}^{\text{elas}} + \sigma_{ij}^{\text{visc}},
\]

where

\[
\sigma_{ij}^{\text{elas}} = \lambda \epsilon_{kk} \delta_{ij} + 2 \mu \epsilon_{ij},
\]

\[
\sigma_{ij}^{\text{visc}} = \lambda' \dot{\epsilon}_{kk} \delta_{ij} + 2 \mu' \dot{\epsilon}_{ij}.
\]
Strain is denoted by $\epsilon_{ij}$ and its time derivative by $\dot{\epsilon}_{ij}$. Tensor notation is used; thus, $\epsilon_{kk} = \epsilon_{11} + \epsilon_{22} + \epsilon_{33}$ and $\delta_{ij} = 1$ if $i = j$ and 0 if $i \neq j$. $\lambda$ and $\mu$ are the Lamé constants and $\lambda'$ and $\mu'$ are the viscosity characteristic parameters. In this paper, the viscosity constant of the medium is defined as $\eta/\Delta$. The Lamé constants can be expressed in terms of the Young’s modulus ($E$) and Poisson’s ratio ($\nu$) of the medium. For the case of an isotropic material, all of the elastic parameters can be defined in terms of only two independent constants of the above.

In most soft tissues, nearly static incompressibility is assumed where $\nu \approx 0.5$ and zero fluid compressibility results in $\lambda' = -\eta/3$ (Malvern 1969). With these assumptions, the matrix representation of (1) is as follows:

$$\sigma = C\epsilon + C'\dot{\epsilon},$$

where vectors $\sigma$ and $\epsilon$ contain the six components of the isotropic stress and strain tensors and $C$ and $C'$ can be derived from equations (1)–(3). $C$ is the material elasticity characteristic matrix (see, for example, Malvern (1969) or Zienkiewicz and Taylor (2000)) and $C'$ is the material viscosity characteristic matrix:

$$C' = \eta/3 \begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}.$$  

2.2. Finite element model of the deformation

If the region of interest is discretized, by requiring the work of external forces to be equal to the work of inertia plus the work of the elastic and viscous forces, the dynamic finite element model of the deformation can be obtained. In this derivation, the internal deformation of the body, which is governed by equation (4), determines the work of the internal viscoelastic forces. The interested reader is referred to Cook et al (1989) for a detailed derivation.

With the excitation $f(t)$ and displacement $u(t)$ for all the nodes as a function of time, the well-known transient FEM model can be obtained as follows:

$$K u(t) + B \dot{u}(t) + M \ddot{u}(t) = f(t),$$

where $K$ is the stiffness matrix which depends on the elements’ elasticity vector $e$, $B$ is the damping matrix and $M$ is the mass matrix composed by globalization of the individual masses (Cook et al 1989). Here, note that $f$ is a vector of the known boundary conditions, including forces and displacements, and therefore it depends on the elasticity distribution (Bro-Nielsen 1998). Usually, $M$ is diagonalized by assuming lumped masses at the nodes. If equation (5) is used in deriving (6), $B$ will be a function of the viscosity vector $\eta$ of the elements. In a general 3D problem with $n$ nodes and $m$ elements, $K$, $B$ and $M$ are $3n \times 3n$ matrices, $e$ and $\eta$ are vectors of size $m$, and $u(t)$ and $f(t)$ are vectors of size $3n$.

The Fourier transform of equation (6) yields

$$(K + j\omega B - \omega^2 M)\hat{u} = \hat{f},$$

where hatted variables denote Fourier transforms and $j$ is the imaginary unit. Decomposing $\hat{u}$ and $\hat{f}$ into their real and imaginary parts, equation (7) can be written as

$$A \begin{bmatrix} \hat{u}' \\ \hat{u}' \end{bmatrix} = \begin{bmatrix} \hat{f}' \\ \hat{f}' \end{bmatrix},$$

where
with
\[ A \triangleq \begin{bmatrix} K - \omega^2 M & -\omega B \\ \omega B & K - \omega^2 M \end{bmatrix}, \tag{9} \]
where the superscripts \( r \) and \( i \) denote the real and imaginary components of a vector, respectively.

3. Inverse problem

Upon applying an external excitation to a soft material, a deformation will be produced within the material, which depends on the mechanical properties, boundary conditions and geometry. A method was proposed in Kallel and Bertrand (1996) to optimize for the tissue Young’s modulus in a finite element model, such that the static least-squares error between measured and model-predicted displacements is minimized. Similar optimization procedures have been studied in a number of other works with static or quasi-static displacement data (Fu et al. 2000, Miga 2003, Oberai et al. 2004, Dooley et al. 2004, Liu et al. 2005, Khalil et al. 2005). The idea is to minimize a functional that is based on the difference between the observed real-valued displacement vector and the predicted one. If, however, a transient or harmonic excitation is applied, the motion will also be affected by dynamic properties according to (6) or (7). Therefore, a more general formulation needs to be derived that takes into account the real and imaginary displacement components and optimizes for the best viscoelastic parameters that match the model.

3.1. Reconstruction method

An inverse algorithm is devised that searches for the optimal values of Young’s modulus and viscosity that yield the least deviation from the observed displacements inside the medium. With the measured displacement vector \( \hat{u}_0 \), an error functional can be defined as follows:
\[ \phi(p) = \frac{1}{2} \| \hat{u}'(p) - \hat{u}'_0 \|^2 + \frac{1}{2} \| \hat{u}'(p) - \hat{u}'_0 \|^2, \tag{10} \]
where \( p = [e^T \eta^T]^T \) is a 2m parameter vector consisting of Young’s moduli and viscosities of the m elements in the model.

The parameter \( p \) can be iteratively updated using the Gauss–Newton algorithm or any descent method. In this paper, the Jacobian has been calculated by differentiating equation (8) and the step direction has been computed with the Levenberg–Marquardt algorithm.

The parameter vector is updated at each iteration as \( p_{k+1} = p_k + \Delta p_k, k \in \mathbb{N} \), where \( k \) is the iteration index. The update vector, \( \Delta p_k \), can be calculated from
\[ H_k \Delta p_k = -J_k^T \Delta \hat{u}_k, \tag{11} \]
where \( J_k \) is the Jacobian, \( H_k \) is the Hessian and \( \Delta \hat{u}_k \) is defined as follows:
\[ \Delta \hat{u}_k = \begin{bmatrix} \Delta \hat{u}'_k \\ \Delta \hat{u}_k \end{bmatrix} = \begin{bmatrix} \hat{u}'_k - \hat{u}'_0 \\ \hat{u}_k - \hat{u}_0 \end{bmatrix}, \tag{12} \]
where, for simplicity, \( \hat{u}_k \) denotes \( \hat{u}(p_k) \). Note that for a 3D FEM mesh with \( n \) nodes, \( \Delta \hat{u}_k \) is a 6m parameter vector.
3.2. Calculation of the Jacobian and the Hessian

The Jacobian matrix describes the sensitivity of the displacements with respect to the parameters, and is defined as follows:

\[
J = \begin{bmatrix}
\frac{\partial \hat{u}_r}{\partial p} \\
\frac{\partial \hat{u}_i}{\partial p}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial \hat{u}_r}{\partial e} \\
\frac{\partial \hat{u}_i}{\partial e} \\
\frac{\partial \hat{u}_r}{\partial \eta} \\
\frac{\partial \hat{u}_i}{\partial \eta}
\end{bmatrix}.
\]  

(13)

Differentiating (8) with respect to the \( j \)th element of \( p \), i.e. \( p_j \), one obtains

\[
\frac{\partial A}{\partial p_j} \begin{bmatrix}
\hat{u}_r \\
\hat{u}_i
\end{bmatrix} + A[J_j] = \begin{bmatrix}
\frac{\partial \hat{f}_r}{\partial p_j} \\
\frac{\partial \hat{f}_i}{\partial p_j}
\end{bmatrix},
\]  

(14)

where \( \{J_j\} \) denotes the \( j \)th column of \( J \) defined in (13). Note that \( p_j \) can be either the elasticity or the viscosity of an element; therefore, \( \partial A/\partial p_j \) can be computed from (9) as follows:

\[
\frac{\partial A}{\partial p_j} = \begin{cases}
\begin{bmatrix}
\frac{\partial K}{\partial e} & 0 \\
0 & \frac{\partial K}{\partial \eta}
\end{bmatrix}, & p_j = e_j \\
\omega \begin{bmatrix}
0 & -\frac{\partial B}{\partial e} \\
\frac{\partial B}{\partial \eta} & 0
\end{bmatrix}, & p_j = \eta_j.
\end{cases}
\]  

(15)

The right-hand side of equation (14) can be calculated by considering the effect of the displacement boundary conditions on the vector \( \hat{f} \). As a result, with known \( A \), equation (14) can be solved for \( \{J_j\} \) and thus the Jacobian matrix can be constructed.

Once the Jacobian has been evaluated at the \( k \)th iteration, the first-order approximation of the Hessian yields

\[
H_k = J_k^T J_k.
\]  

(16)

Note that this modified Hessian ignores the effect of the second-order residuals in the least-squares problem of (10). However, in the proximity of the solution, this error is negligible (Nocedal and Wright 2006).

In general, with \( n \) nodes and \( m \) elements with unknown parameters in the 3D mesh, \( J \) is a \( 6n \times 2m \) matrix and \( H \) is a \( 2m \times 2m \) matrix.

3.3. Practical considerations

Equation (11) requires the Hessian to be invertible. However, when the Jacobian is rank deficient, \( H_k \) in (16) will be badly conditioned and may not be easy to invert. The Levenberg–Marquardt algorithm modifies the Hessian in such a way that the resulting step will be a combination of the directions predicted by the steepest descent and the Gauss–Newton methods, depending on how far the current point is from a local minimum. If far from a local minimum, the algorithm behaves like the steepest descent method. However, in the vicinity of a minimum, the step direction will be close to that of the Gauss–Newton method and the positive definiteness of the Hessian can be guaranteed (Marquardt 1963, Nocedal and Wright 2006). The Hessian may thus be modified as follows:

\[
H_k = J_k^T J_k + \lambda_k I_{2m \times 2m},
\]  

(17)

where \( I_{2m \times 2m} \) is an identity matrix with \( m \) being the number of elements with unknown parameters. \( \lambda_k \) is a small regularization factor that makes \( H_k \) positive definite. This method
Viscoelastic characterization of soft tissue from dynamic finite element models has been reportedly used with success in the context of the inverse problem of elasticity (e.g. Kallel and Bertrand 1996). The required update vector can be obtained by utilizing a trust-region or line-search procedure (Nocedal and Wright 2006).

The reconstruction result is highly dependent on the displacement noise. As suggested by Doyley et al (2000), a spatial filter can be applied to the modulus distribution at every iteration to make the solution of the problem smooth. As a result, the modulus of each element will be a weighted sum of its own value and the moduli of its adjacent elements. A linear filter can thus be constructed in the form of a sparse matrix that contains the required weights for each element. The filter can be convolved with the updated modulus distribution at each iteration to conduct the optimization toward a smooth solution.

To constrain the problem and avoid physically infeasible or implausible results, assumptions can be made on the parameters by adding constraints to (10). In particular, negative elasticity and viscosity values should be avoided. To bound a parameter \( p \) to values greater than or equal to \( p_L \), a technique based on the change of variables can be implemented. Therefore, an auxiliary variable \( s \) can be defined such that \( p = s^2 + p_L \). Using the chain rule, the problem can be formulated and solved for \( s \), and finally \( p \) can be calculated from the optimized value of \( s \).

To ensure a sufficient decrease of the functional at every iteration and guarantee convergence, a line search procedure has been implemented to determine the step size based on the Armijo condition, which uses a first-order approximation of the functional (Nocedal and Wright 2006).

4. Tests

4.1. Two-dimensional modeling

4.1.1. Plane stress versus plane strain. While a 3D model accounts for the deformations in all directions, certain assumptions can be made to simplify the problem to only two dimensions. When the medium is confined such that the out-of-plane strain and displacements are minimized, a plane-strain model is used to analyze the deformations. This happens when some walls parallel to the imaging plane confine the medium or when the thickness of the medium is significantly larger in the elevational direction. Also, if the axial compression is uniformly applied to the top surface of the medium, a plane-strain assumption may be valid on the middle plane (Steele et al 2000). On the other hand, a plane-stress assumption can be legitimately made when the elevational extent of the medium is much smaller than the other two dimensions.

In this work, for error quantification and performance analysis, a 2D plane-stress case is assumed. Under this assumption, the material characteristic matrices for the elasticity and viscosity can be computed by setting the out-of-plane components of the elastic and viscous stress tensors equal to zero. This results in the following 2D characteristic matrices:

\[
C = \frac{E}{(1 - \nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{(1-\nu)}{2} \end{bmatrix},
\]

\[
C' = \frac{\eta}{2} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix},
\]

where \( E, \eta \) and \( \nu \) are Young’s modulus, viscosity and Poisson’s ratio, respectively. Alternatively, under the plane-strain assumption, the characteristic matrices change to the
following ones:

\[
C = \frac{E}{(1 + \nu)(1 - 2\nu)} \begin{bmatrix} (1 - \nu) & \nu & 0 \\ \nu & (1 - \nu) & 0 \\ 0 & 0 & (1 - 2\nu) \end{bmatrix},
\]  
\[
C' = \frac{\eta}{3} \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.
\]  

4.1.2. Using only axial displacements. If the lateral displacements are not available, the functional in equation (10) can be evaluated in terms of the axial data only. Therefore, once the Jacobian has been calculated, the rows that correspond to the lateral data can be removed and then the step can be evaluated from equation (11). In the following simulations and experimental analysis, only axial displacements were used in the inverse problem and the elasticity and viscosity at the top of the phantom were assumed constant, thus constraining the optimization. The lateral displacement and force distribution were not used in solving the inverse problem. The block diagram for the procedure to solve the inverse problem in the simulation and experiments is depicted in figure 1. A stopping criterion monitoring the gradient size and the number of iterations has been used. Three criteria were checked in order to stop the iterations, including the value of the functional, the functional gradient and the maximum number of iterations. The thresholds for these conditions were chosen by trial and error.

4.2. Numerical simulations

A 2D 4 cm × 4 cm region, bounded from above with laterally nonslip conditions, has been meshed with four-node rectangular elements. Models of the elasticity and viscosity, as defined in equations (18) and (19), were adopted. Figure 2 shows a typical finite element grid as used in the simulations. As can be seen, a rectangular region of interest is bounded at the top and excited from below by applying force or displacements to one or several nodes. In the plane-stress formulation, the thickness of the region has been assumed to be 1 cm. Subject to a force or displacement excitation, the nodal deformation could be calculated as explained in section 2.2. With a static displacement excitation, it has been shown that the solution to the inverse problem of elasticity in the plane-strain state is unique, given the value of Young’s moduli on the lower or upper boundaries of the medium (Barbone and Bamber 2002). The uniqueness of the inverse problem of viscoelasticity under plane-stress dynamic deformation has not been explicitly studied in the literature. It is possible that the same plane-strain analysis of Barbone and Bamber (2002) can be performed for the plane-stress case to obtain similar uniqueness criteria with complex-valued parameters as in our case. We proceed with the assumption that a unique solution can be obtained by knowing the viscosity and elasticity on the top row of the finite element grid, given only the complex axial displacements.

A constant density of 1000 kg m⁻³ is assumed for the medium which is typical of soft tissues. Within constant regions, Young’s modulus and viscosity are assumed to be 10 kPa and 10 Pas, respectively, which are chosen arbitrarily in the range of the viscoelastic properties of human soft tissues. Poisson’s ratio has been assumed to be 0.495 to mimic the near incompressibility of soft tissues.

To elucidate the significance of modulus imaging versus interpretation of the strain images, a medium with an elasticity and a viscosity inclusion has been simulated. Figure 3(a) shows
the locations of the two inclusions. The background has Young’s modulus of 10 kPa and a viscosity of 10 Pas, while in the elasticity inclusion Young’s modulus is 30 kPa and the viscosity of the viscous inclusion is 30 Pas. A symmetric mesh of $21 \times 21$ nodes has been applied to the region and a 5 Hz steady-state compressional displacement excitation has been applied to the phantom in the axial direction. In a first trial, all of the bottom nodes of the phantom were excited and the resulting axial strains were computed. Figures 3(b) and (c) show the real and imaginary parts of the axial strain, respectively. In the second test, only five middle nodes at the bottom of the grid were excited, which resulted in the strain images in figures 3(d) and (e). One can note a high dependence of the strain images on the loading and boundary conditions. The presence of artifacts in the images makes it hard to delineate the inclusions by direct interpretation of the strain images; however, solving an inverse problem with the knowledge of the boundary conditions and the loading profile enables one to reconstruct the modulus distributions accurately. Using the displacements under the first described loading conditions, the reconstructed images are shown in figures 3(f) and (g) for Young’s modulus and viscosity respectively. Due to zero noise in this simulation, the parameters did not need smoothing;

![Figure 1. Block diagram of the optimization procedure.](image-url)
Figure 2. 2D finite element model, using rectangular elements. The region is bounded at the top and excited from below by a force or displacement distribution of a known frequency. Note that the arrows do not specify the nodes that are excited, but the presence of a distributed or localized excitation. The axial and lateral directions of the ultrasound are shown.

therefore, perfect reconstruction can be observed. Similar images could be obtained using the data from the second loading condition.

The sensitivity of the estimations to the displacement noise is illustrated in figure 4. The same material properties and inclusion locations as in figure 3(a) have been used and the same mesh with $21 \times 21$ nodes has been applied to the region. The entire lower side of the region was subjected to a 5 Hz steady-state excitation. White noise was then added to the resulting displacements and the inverse problem was solved for the elasticity and viscosity values. Two criteria were utilized to assess the performance of the estimation. First, the RMS error was measured based on the difference between estimated moduli and the actual ones. The RMS error for the elasticity can be defined as follows:

$$
\epsilon(e) = \sqrt{\frac{1}{m} \sum_{i=1}^{m} \frac{(e_i^e - e_i^0)^2}{(e_i^0)^2}},
$$

(22)

where $m$ is the number of the elements and $e_i^0$ and $e_i^e$ are the actual and estimated elasticities for the $i$th element, respectively. The RMS error for the viscosity is defined in the same way. For each level of SNR, the model was simulated 50 times and the means and standard deviations of the RMS error for elasticity and viscosity are shown in figures 4(a) and (b) respectively. If instead of using the raw noisy displacements, a small $3 \times 3$ averaging filter is applied to the displacements prior to running the inverse algorithm, a significant improvement can be achieved in lower displacement SNR values (around 40 dB), while at high SNRs, the smoothing results in poor estimation at the boundaries of the inclusion and thus a higher RMS error.

As a second criterion to analyze the exactness of the solution, the contrast-to-noise ratio (CNR) of the estimated elasticity and viscosity images with inclusions can be used (Varghese and Ophir 1998). If the area of interest is composed of two homogeneous regions with different Young’s moduli, the CNR($e$) can be defined as follows:

$$
\text{CNR}(e) = \frac{2(s_1 - s_2)^2}{\sigma_1^2 + \sigma_2^2},
$$

(23)
Figure 3. The axial strain around an elasticity and viscosity inclusion. The locations of the two inclusions are shown in (a). The maps of the real and imaginary parts of the axial strain due to a 5 Hz harmonic displacement excitation of the lower surface are shown in (b) and (c). The real and imaginary axial strain profiles due to applying the same excitation to only a few nodes at the middle bottom of the phantom are shown in (d) and (e) respectively. A 2D dynamic finite element model has been used to generate the strain fields. The artifacts that are seen in the strain images are caused by several factors including the loading profile, boundary conditions and geometry. By solving the inverse problem, the elasticity and viscosity inclusions can be accurately delineated as illustrated in (f) and (g) respectively. The inverse problem was solved for the case where force was exerted along the whole lower surface (from (b) and (c)), but similar results are obtained with the data from the other case.

where $s_1$ and $s_2$ are the average elasticities estimated at the inclusion and at the background and $\sigma_1$ and $\sigma_2$ are the standard deviations of the elasticity estimates in those regions respectively.
Figure 4. The displacement SNR changes the accuracy of the estimation. The computed displacements have been corrupted by a known power of noise and the parameters have been estimated using the raw data (the results are in black solid lines) or spatially filtered displacements (the results are in gray dashed lines). The RMS error for the elasticity and viscosity estimates are shown in (a) and (b) respectively, while the CNR for those estimates are depicted in (c) and (d).

CNR(\eta) can also be defined for the viscosity estimates in a similar way. A high CNR value will be obtained if the estimates are smooth in each region but quite discriminated between the two. The means and standard deviations of the CNR values versus the displacement SNR for the aforementioned phantom are plotted in figures 4(c) and (d) for the elasticity and viscosity, respectively. It can be seen that the contrast is enhanced when the displacements are filtered prior to solving the inverse problem at higher noise levels, while the contrast is compromised at higher SNR values.

If the lateral displacements and forces at some or all parts of the boundaries are not available, some artifacts may be present in the reconstructed parameters. The same medium as depicted in figure 3(a) has been embedded in the middle of a larger region with the same background properties. The forward problem has been solved for a 6 cm x 6 cm region. As in the previous case, the lower end of the phantom was excited by a 5 Hz axial harmonic motion, while the top was held stationary. The region of interest (ROI) was the middle 4 cm x 4 cm of the phantom that enclosed the two inclusions as shown in figure 5. The parameters have been reconstructed using only the axial displacements in the ROI and with different assumptions on the lateral motion at the boundaries. Figure 6 illustrates the effect of unknown boundary conditions on the reconstruction results. Spatial filtering has not been used on the estimated parameters in order to obtain a clear comparison. First, zero lateral force has been assumed on the boundaries. The reconstructed images of the elasticity and viscosity are shown in
Figure 5. Solving the inverse problem for a ROI with unknown lateral boundary conditions. A 6 cm × 6 cm region has been simulated while only the axial displacement at a 4 cm × 4 cm region in the middle has been used for parameter identification.

Figure 6. The effect of unknown boundary conditions on the reconstruction of elasticity and viscosity. The medium in figure 5 has been simulated. The axial displacements were used to solve the inverse problem in the illustrated ROI. Different lateral boundary conditions were assumed and the inverse problem was solved: no lateral boundary forces were assumed in (a) and (d), the top and the bottom of the ROI were assumed not to move laterally in (b) and (e), and all four sides of the ROI were assumed to have zero lateral motion in (c) and (f). Note that the simulations were performed for an FEM model with 21 × 21 nodes, while the displayed images are up-sampled by a factor of 5 for better illustration.

figures 6(a) and (d), respectively. Next, the top and the bottom of the region have been assumed to have zero lateral motion, while the right and the left boundaries were free to move
lateral. The results are shown in figures 6(b) and (e). Finally, all four sides of the region were assumed to have zero lateral motion and the results are depicted in figures 6(c) and (f).

Another source of error in solving the inverse problem of viscoelasticity arises from the fact that the deformation of the body is three dimensional. Plane-stress or plane-strain assumptions are intended to simplify the model to 2D; however, such simplifications may result in inaccurate estimations. To demonstrate this, a 3D region (4 cm × 4 cm × 3 cm) has been simulated with the same properties and inclusions as denoted before. Having the top of the phantom axially fixed and free to move in the lateral direction, a 10 Hz displacement excitation has been applied to the bottom surface in the axial direction. The forward problem was solved using equation (7). As illustrated in figure 7(a), the axial displacements in the middle plane were used to reconstruct the elasticity and viscosity using the proposed method. The reconstruction was performed once with the plane-stress assumption (figures 7(b) and (d)) and another time using plane-strain matrices (figures 7(c) and (e)). The results show that a 2D plane-stress approximation can be used to model such a geometry; however, the artifacts can be significantly reduced if a plane-strain assumption is applied.

Besides the 2D simplification of 3D deformations, a significant error may arise from using an inaccurate model in the procedure. To show how the reconstruction result may be affected if a different structure of the damping matrix is assumed, the test with a 5 Hz distributed displacement excitation on the bottom of the phantom has been repeated using the same geometry and inclusions as before. To simulate the displacements at the first stage, the damping matrix has been constructed in a way similar to the stiffness matrix, i.e. equation (18) was used instead of (19), with a multiple of the viscosity substituting $E_0$. This is in accordance with Rayleigh damping where the damping matrix of every element is proportional to its stiffness matrix (Semblat 1997). This model of damping, however, has not been taken into account in the reconstruction; thus, the original damping structure as it results from equation (19) has been used in the inverse procedure. The estimates, as shown in figure 8, indicate that the inclusions can still be identified; however, the presence of artifacts in the viscosity image is noted as a consequence of improper model assumptions. Some correlation can also be noticed between the artifacts in the viscosity image and the elasticity distribution.

4.3. Experiments

A tissue mimicking phantom has been constructed with 12% (by weight) bovine skin gelatin in water. 2% cellulose (by weight) has been uniformly added to the phantom prior to solidification to act as ultrasound scatterers and then the mixture was poured in a 38(x) × 40(y) × 25(z) mm³ mold. A small piece of a polyvinyl acrylic (PVA) sponge (Ceiba Technologies, Chandler, AZ, USA) was embedded in the phantom shortly before solidification. The sponge was soaked in water and degassed. This type of PVA sponge has relatively high Young’s modulus compared to its background gelatin and a significantly higher relaxation time and viscosity (Eskandari et al 2008). The phantom was placed on a specially designed shaker (Eskandari et al 2008), and ultrasound RF data were captured. The axial and lateral motions of the phantom at the transducer side were nearly zero while the axial displacement of the other side was estimated. Since the phantom was ultrasonically imaged up to a depth of 37 mm (before the bottom of the phantom), the lateral motion at the bottom of the phantom is not known. It was assumed that lateral force on the phantom was insignificant. A wide-band displacement excitation (1–30 Hz) with a Gaussian amplitude distribution with a standard deviation of 44 μm was applied to the bottom of the phantom. The internal motion of the phantom was estimated using a cross-correlation-based algorithm with prior estimates (Zahiri-Azar and Salcudean 2006). Frequency analysis was performed on the displacements and the real and imaginary
Figure 7. The effect of the 3D deformations on the estimation when a 2D model is used to reconstruct the parameters. (a) The 3D phantom that has been simulated with a high elasticity and a high viscosity inclusion is shown. The axial displacements in the middle plane were used to solve the inverse problem in 2D. The reconstructed elasticity and viscosity with the plane-stress assumption are shown in (b) and (d), while having a plane-strain assumption produces elasticity and viscosity estimates in (c) and (e), respectively.
Figure 8. The effect of using an inaccurate model in the inverse problem for (a) the elasticity and (b) the viscosity estimations. The displacements were computed using a model of the damping in which the damping matrix of every element is assumed to be proportional to its stiffness matrix. The inverse problem was then solved using the proposed damping structure as it results from equation (19).

parts at 10.7 Hz were selected to solve the inverse problem. Since power spectral analysis yields spectral data at discrete frequencies depending on the window length that is applied to the time-domain data (Ljung 1999), only data at certain frequencies were available to solve the inverse problem. Therefore, displacements at 10.7 Hz were chosen arbitrarily in the frequency range of excitation, such that considerable viscoelastic behavior could be observed in the phantom.

In conventional ultrasound, an image is acquired by collecting data from every line, progressively. Hence, there is a systematic delay between the data recorded from different lines of the image, which causes a linear phase difference between the displacements at different lines. As a pre-processing stage on the experimental data, this linear lateral phase gradient due to the limitation of the acquisition time between different lines was calculated and its effect was removed. For this reason, the phase of the displacements was measured and a linearly increasing trend was optimally identified in their lateral profiles. By removing this trend, the acquisition delays were compensated. The displacements were originally estimated in a grid of 49 lines by 67 blocks, filtered and then down-sampled by a factor of 2. As shown in figure 4, filtering the displacements improves the accuracy of the estimation at the range of SNR that is typical for elastographic motion estimation. The down-sampling was performed to reduce the number of variables and the size of the problem. This was only achieved by trading off spatial resolution with processing speed and system memory.

Based on independent rheometric measurements, the background gelatin had Young’s modulus and a relaxation time of approximately 15 kPa and 1 ms respectively. The relaxation time is the ratio of the viscosity to Young’s modulus of the material. The relaxation time of the PVA sponge was also measured to be approximately 4.5 ms at 10.7 Hz (Eskandari et al. 2008). Using initial conditions of 15 kPa and 15 Pas for the elasticity and viscosity for the entire medium, the inverse problem was solved and the parameters were reconstructed. Since the elevational size of the phantom was nearly one-half of the other two dimensions, a plane-stress model was presumably more accurate than a plane-strain one in describing the internal deformations. The iterative scheme was set to stop after the gradient reached a sufficiently small value (Nocedal and Wright 2006). The B-mode image and approximate location of the inclusion are depicted in figure 9(a).
Figure 9. (a) B-mode image of the gelatin phantom with a small PVA sponge inclusion. The approximate location of the inclusion is delineated in the image. The results of (b) Young’s modulus and (c) viscosity estimations are shown. The relaxation-time image (d) is produced as the ratio of viscosity to Young’s modulus. The location of the inclusion and the relative values of the parameters inside and outside the inclusion are consistent with the B-mode observations and rheometry measurements. The artifacts in the images may be associated with the lack of information about the lateral boundary conditions. The images have been filtered and up-sampled for a better illustration.

(This figure is in colour only in the electronic version)

The reconstructed images of elasticity and viscosity are shown in figures 9(b) and (c), respectively. The estimated relaxation time is also shown in figure 9(d). It can be seen from the reconstruction results that the relaxation-time difference between the medium and the inclusion is approximately 2 ms. This parameter was measured to be 3.5 ms in rheometry tests. Although some artifacts are present in the estimated images, the inclusion can be clearly distinguished from the background material.

5. Discussion

The feasibility of reconstructing the elasticity and viscosity in a non-homogeneous medium based on a finite element analysis has been studied in this work. A model has been devised to account for the viscoelastic changes in soft tissues. The simulations and experiments were
performed in 2D while they can be generalized to a 3D problem if other components of the
displacements can also be measured experimentally. The assumption of a nearly plane-stress
deformation has been made in this work and the simulations and inversion techniques have
been utilized accordingly. If, however, a plane-strain state is known to explain the observed
deformations more accurately, the same inverse algorithm can be used with a proper plane-
strain formulation of the forward problem. While 2D FEM is a simplification of the more
general 3D problem, even the proposed 3D formulation or other 3D models in the literature
may not accurately model the deformations.

As explained in section 2.2, the mass matrix in dynamic finite element analysis can be
diagonalized to improve computational efficiency. A diagonal representation of the damping
matrix is also permissible in some situations. A more general scheme to characterize the
damping matrix, coined as Rayleigh or proportional damping, is to form it as a linear
combination of the stiffness and mass matrices (Semblat 1997). Depending on the structure and
frequency, the effect of either stiffness or mass matrices may be dominant, which respectively
make the damping matrix approximate a consistent or lumped representation. The damping
matrix using this approach is frequency dependent, where the part attributable to the stiffness
matrix increases with increasing frequency and the part attributable to the mass matrix increases
with decreasing frequency (Cook 1989). A more realistic characterization of the damping
matrix for soft viscoelastic materials has been proposed in this paper. The equations of dynamic
equilibrium have been discretized in a finite element mesh and further simplified to obtain the
desired relationship between the force and displacement vectors. With this representation, one
can be sure that within the discretization error, the acquired finite element model agrees with
other viscoelastic formulations based on the equilibrium equations.

To reduce the condition number of the Hessian matrix (often larger than $10^{12}$), two
methods have been tested. In the Levenberg–Marquardt method, the step is close to that of the
Gauss–Newton when the Hessian is invertible and similar to that of the steepest descent when
the Hessian is non-invertible. Another technique is to modify the small eigenvalues of the
problem to limit the condition number of the Hessian with an upper bound. In the simulations
and experiments in this paper, the difference between the results from the two methods was
seen to be insignificant; thus, the Levenberg–Marquardt technique has been selected.

The poor condition numbers of the inverse problems in elasticity and viscosity make
the final solution dependent on the initial conditions and the optimization technique that is
used. One approach to obtain a smooth profile of the parameters is to add the Laplacian of the
parameters to the functional and search for the solution that minimizes the displacement
difference and the Laplacian term at the same time (Oberai et al 2004). In this paper, as
suggested in Doyley et al (2000) the parameters were spatially filtered by a $3 \times 3$ averaging
kernel at each iteration to render a smooth solution when dealing with noisy displacements.

The inverse problem of elasticity and viscosity is solved using only axial displacements.
Figure 3 demonstrates the artifacts in the strain images and how they can be removed by
solving the inverse problem. While knowledge of the lateral boundary conditions will result
in accurate estimation and removal of the artifacts, from figure 6 it can be seen that axial data
are not sufficient to characterize the medium. With lateral motion estimation in elastography,
axial and lateral displacements can both be used to remove the artifacts corresponding to
unknown boundary conditions.

The simulations were performed with a relatively coarse mesh of $21 \times 21$ within a $40 \times
40$ mm$^2$ region. The spatial resolution can be enhanced by using a finer grid size, at the expense
of computational resources. Alternatively, spatial resolution can be enhanced recursively
through mesh refining. For this purpose, a coarse mesh can be first applied to the original
problem. Once the parameters have been reconstructed, the resolution can be enhanced in
different regions of interest by applying a finer localized mesh, so that the inverse problem can be solved with higher spatial resolution only in that region. Based on figures 5 and 6, such a refining window can be applied to any part of the image, given that the lateral boundary conditions are partially known for that window.

The error in estimation has been measured by defining two criteria: RMS error and contrast-to-noise ratio. Figure 4 shows that below a certain level of displacement SNR, parameter reconstruction results in noisy and unreliable estimates. This sensitivity to noise may be lowered by appropriate filtering of the noisy displacement data. The RMS error seems to reach a plateau rather than converging to zero at high SNR values. This is due to filtering of the parameters at every iteration as explained in section 3.3. Based on equation (22), the RMS error may originate from either estimation bias or variance (Ophir et al 1999, Fromageau et al 2007). The errors quantified in figure 4 for simulated data are mainly due to the latter, which indicates the lack of precision in the estimates. Compared to the estimation variance, the bias can be made small by choosing suitable initial conditions for the recursions and assuming correct parameters as the constrained boundary values for the problem. Both of these errors are important factors in producing reliable images; however, the presence of a small amount of bias in the estimated parameters may still yield decipherable images while estimation variance may drastically drown out the modulus distribution.

In vivo situations, it may not be feasible to know the exact boundary conditions of the tissue. In that case, lateral displacement measurements would help reduce the artifacts in the reconstructed parameters. However, due to the coarse lateral resolution of ultrasound imaging, if accurate lateral motion estimation is not obtainable, some basic assumptions should be made on the lateral force or displacement on the boundaries of the imaging window. Figure 6 shows how such assumptions influence the reconstructions. In this example, uncertainty of the lateral boundary conditions had negligible effect on the estimated Young’s modulus, while estimated viscosity is seen to be dependent on the assumed boundary conditions. In this particular case, one can see that, in comparison with the other assumptions, a zero force assumption produced a more acceptable viscosity estimate in figure 6(d) while a faded shadow of the elasticity inclusion is visible. Although accurate knowledge of the lateral boundary conditions is necessary, it is not sufficient to produce estimations that are free of artifacts. As seen in figure 7, simplifying a 3D problem to a 2D one can also produce errors. In the case of figure 7(a), the elevational dimension of the phantom was comparable to the other dimensions; hence, a plane-strain model was relatively more accurate in describing the deformations. Here, a plane-strain model would be best suited if the thickness of the phantom were increased, while a smaller thickness would make the result of a plane-stress model more reliable.

The systematic estimation error caused by choosing inaccurate models is depicted in figures 7 and 8. Ideally, the reconstruction results in both of these cases should be the same as those in figures 3(f) and (g). Using a 2D model to describe the 3D deformations will be acceptable only if the dimensionality reduction to obtain a plane-stress or plane-strain state is reasonable. The blurs and artifacts in figures 7(b) and (d) are due to the marginal accuracy of the plane-stress approximation. In this case, a plane-strain model produced less artifact and more clear images as seen in figures 7(b) and (d). Also, based on the findings in figure 8, one should expect to see some artifacts from misrepresentation of the damping phenomenon, since the model proposed by incorporating equation (5) or (19) into the FEM equations may not accurately describe the damping mechanisms in soft tissues.

The algorithms have been tested with experimental data. A gelatin-based phantom was constructed with a hard and more viscous sponge-like PVA inclusion. The estimation results are in agreement with the rheometry tests reported in Eskandari et al (2008). However, rheological models do not account for tissue compressibility and compressibility was not
considered in the FEM model either. During phantom construction, the small piece of sponge was degassed and saturated with water, so that the volumetric changes are likely to be dominated by water. Since the volume within the inclusion was preserved with the water trapped inside, a nearly incompressible condition has been assumed. To what extent this was achieved is an acknowledged challenge. The porosity of the sponge material may make it compressible and this would introduce errors in the parameter identification. However, in such an FEM model, it is only the viscoelasticity that can account for the observed phase changes, while porosity and compressibility result in a change in the static deformation profile.

Lateral forces were not measured at the boundaries, and maintaining non-slip lateral conditions at the boundaries is not trivial. Therefore, unknown lateral boundary forces and displacements in the problem produced artifacts in the reconstructed images as expected from the simulations. The gray shadows below the inclusion in both images of figure 9 are due to the lack of information about the boundary conditions as well as inaccuracy in estimating the displacement phasors. Due to a lower sonographic SNR in the region below the sponge, relatively poorer displacement estimates were obtained, which also lowered the estimation accuracy in that region compared to the rest of the phantom. If lateral displacements are measured or lateral forces are known at the boundaries, accurate information can be used to enhance the reconstruction results. In addition, a plane-stress assumption has been made in this experiment due to the relatively smaller elevational dimension of the phantom compared to the other dimensions. While plane stress should be a more suitable model than plane strain, it would be more accurate if the thickness were smaller. Therefore, some systematic error should be expected due to utilization of a simplified model.

In in vivo experiments where boundary conditions are not known, high quality estimates of the axial and lateral displacements are necessary to solve the inverse problem in any region of interest. Given the much more complex environment when dealing with human subjects, knowledge of the lateral displacements may be crucial to decouple the effect of the boundary conditions. However, for simple situations where legitimate assumptions can be made for the lateral displacements, the axial component of the motion may be sufficient to reconstruct the viscoelastic properties.

6. Conclusion

The feasibility of measuring the viscoelastic parameters of soft tissue using dynamic finite element models has been explored in this paper. A finite element model has been derived to account for the viscoelastic changes in the soft tissue, consistent with the known rheological models. Using this deformation model and the observed dynamic data, the viscosity and elasticity have been reconstructed in the medium by solving an inverse problem. The proposed algorithm was derived for the general 3D finite element model. The 2D algorithms were also explored to comply with conventional ultrasound elastography. It is known that the solution to the elasticity inverse problem using quasi-static FEM is subject to high noise sensitivity. The accuracy of estimating the viscoelastic parameters using the proposed algorithm was found to be dependent on the boundary conditions, displacement noise level and appropriate temporal and spatial filtering of the displacements. An iterative solution to the inverse problem of elasticity and viscosity has been formulated. A well-known Gauss–Newton-based approach to solve the inverse problem of elasticity has been modified in such a way that steady-state harmonic deformations are used to estimate the viscoelastic properties, and the need for lateral information is minimized. The solution can be computed by incorporating a priori knowledge of the moduli, such as the values of the parameters on the boundaries and the smoothness of the solution. The methods were studied in FEM simulations, and the effects of the displacement
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Noise and boundary conditions were explored. The approach has been tested in an experiment and has successfully reconstructed the viscosity and elasticity in a gelatin-based phantom with an inclusion of known properties. It was found that in some simple situations, only axial displacements and approximate knowledge of the lateral conditions at the boundaries may be sufficient to characterize the medium. In more complex cases, however, utilizing both axial and lateral displacements at the boundaries may be needed to successfully estimate all the parameters.

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